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Use of Calculators in High School Mathematics

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Abstract

For many collegiate level mathematics courses at universities across the country, students are not allowed to use calculators. This is often a huge adjustment for students who have spent their entire high school career using calculators. Whether it is graphing or simple arithmetic, students relied on calculators in high school, but for college, the tool of a calculator may not be available for them. This leads to a multitude of questions: Should calculators be used in high school? Do calculators help students learn or are they simply a tool to get answers? Are calculators beneficial in some courses, but not others? These are the questions that will be explored in this paper. From research, interviews, and surveys, this paper will allow current and future teachers to reflect on their opinion of the use of calculators in the classroom at a high school level.

Introduction

As an Adolescent to Young Adult Integrated Mathematics major (working towards a grade 7-12 license), I am around mathematics every single day. I have taken numerous college courses from Calculus to Abstract Algebra. I have conducted observation hours at Buchtel High School, Akron East Community Learning Center, and Hartford Middle School. I have been a tutor for fellow college students at the University of Akron for two years in College Algebra, Pre-Calculus, and Calculus. In every single one of these experiences, I have noticed a theme: the desire to use a calculator. In my own courses at the University of Akron, a question that is always asked on the first day is "Are calculators allowed?". In the high school and middle school classes that I have observed, students always have calculators on them and often times as an application on their phone that they rely upon greatly. As a tutor, I have heard numerous complaints about professors not allowing them to use a calculator which leads them to make simple arithmetic mistakes. All of these instances tend to show an overreliance on calculators which could limit student learning.

When I was observing a sophomore geometry class at Akron East Community Learning Center, the teacher who I was observing had the students research what they believed was the greatest tool in the history of mathematics. Not to my surprise, one group claimed that a calculator was the greatest tool in history because "it helps solves big problems." I then approached the group and asked them, "Do you believe calculators help you learn math?" All three of them gave a resounding, "No!". Because of this and the reasons stated above, I think it is important to research calculator use in the classrooms. As a future teacher, I want to maximize student learning in any way that I can. I decided to research this topic so I can come to a

conclusion on how much I will allow students to use calculators in my classroom. Hopefully, this research will enable other teachers, future and current, to consider it as well.

History of Calculators

One of the earliest calculating tools for performing arithmetic was the abacus. This tool is often made with a wooden frame with beads sliding back and forth on sliders. The very first abacus was more than likely on a flat stone that was covered in sand. Words and letters were drawn in the sand and eventually numbers were added along with stones to aid the calculation process. This dust form of an abacus was used by the Babylonians around 2400 B.C.E., while the abacus with counters and strings was founded many years later with India, Mesopotamia, and Egypt being potential locations for its origin (Abacus - New World Encyclopedia, n.d.). A picture of an abacus can be seen below (Abacus - New World Encyclopedia, n.d.).

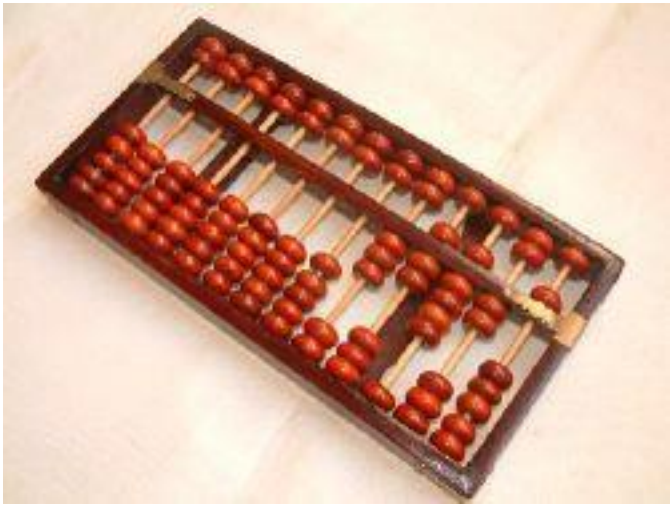

Figure 1: Example of an abacus

The mechanical calculator was created in 1623, when German mathematician and astronomer, Wilhelm Schickard, built the first known calculator which he called the Calculating Clock. He wrote a letter to fellow astronomer Johannes Kepler explaining his machine and the capabilities

of it (Swaine & Freiberger, 2016). Schikard built two versions of the Calculating Clock. The first, which was built for Kepler, was destroyed in a fire, and the second has been lost or destroyed over time (Computer History Museum, n.d.). A replica of the Calculating Clock can be seen below (Computer History Museum, n.d.).

Figure 2: Replica of Calculating Clock

Almost twenty years after Schickard made the first real strides toward what the calculator is known as today, the French mathematician Blaise Pascal made even greater progress. In 1639, Pascal's father took a job as a tax collector. Two years later Pascal created a mechanical calculator, called the Pascaline, to help his father in his career. The machine could have been much more successful, but the French money system provided many challenges as there were 20 sols in a livre and 12 deniers in a sol. Had it had been out of factors of 100, it would have made it much easier for Pascal. However, the Pascaline was fairly accurate as it handled addition and subtraction. Multiplication and division had to be done through a series of additions and subtractions. Pascal worked on improvements to the machine for a couple of years and sold a few versions before manufacturing stopped ten years after he created it (MIT Program, n.d.). An image of a replica of the Pascaline can be seen below (Computer History Museum, n.d.).

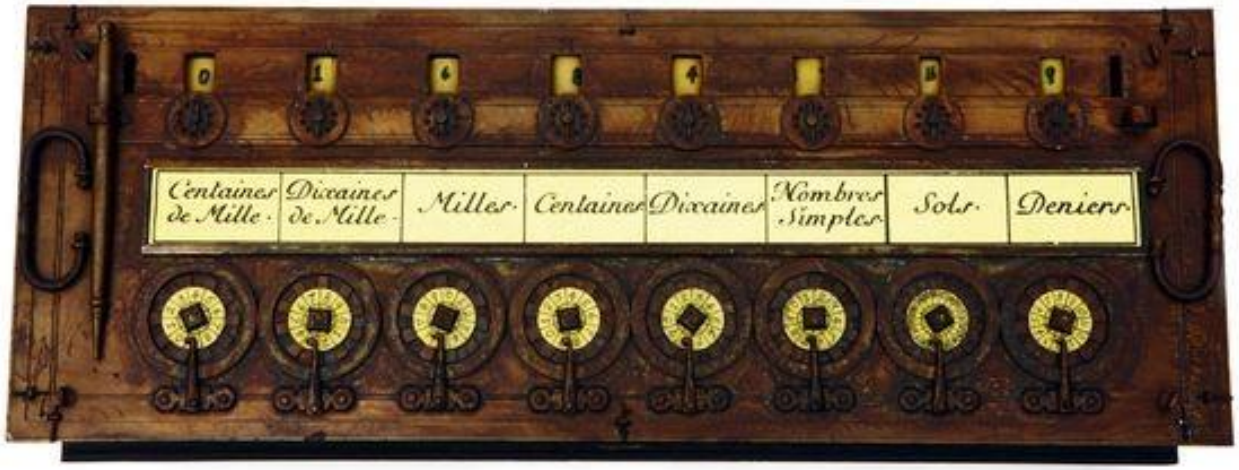

Figure 3: Replica of Pascaline

During the 1800's and specifically the 1900's, the advancement of calculators took a huge leap. But it wasn't until the 1970's when calculators truly transformed the way millions of people performed arithmetic. Handheld calculators found its way to engineers, businesses, and even consumers. In education, many educators were starting to question how much students should even learn about the procedures in multiplication, division, and taking square roots (Smithsonian Institution, n.d.).

Companies started to release calculators to the market at quite the expensive rate. Hewlett-Packard Corporation came out with the HP-35 scientific calculator in 1972 that could add, subtract, multiply, divide, and solve trigonometric functions. The HP-35 was sold on the marketplace for $395. Texas Instruments responded to the HP-35 with the TI-50 for a cost of only $150 (Smithsonian Institution, n.d.). There were other calculators on the market that were much cheaper, but they were just four function calculators. Price ranges for these Japan-made calculators were in the range of $5.95 - $19.95 (Smithsonian Institution, n.d.).

Today, one of the most common graphing calculators found in classrooms across America is the TI (Texas Instrument) 84. This calculator has many advanced functions. It can graph functions, store lists, solve definite integrals, solve matrix operations, solve statistic operations such as mean, median, and variance, can plot scatter plots and histograms, and can work with chi-square, binomial, and poisson functions. This isn't even the full list as it is capable of so much more. The TI-84 is even approved for the PSAT, SAT, and ACT (84 Plus CE Graphing Calculator, n.d.). This specific calculator can be bought for around $100 on various websites and stores. It has become a great tool for students in the classroom and also for professionals in their work environment.

The advancement of calculators over time has greatly influenced what mathematicians have been able to accomplish. They have not only impacted the mathematical field, but they have impacted other fields such as engineering, accounting, and economics as people in these professions use calculators on a day to day basis. They have also helped businesses and their speed with monetary transactions. With technological advances that have happened over the past half century, almost every person in the United States now has access to a calculator either as a handheld, application on computer, or application on their phone.

Pros and Cons of Calculator Use

Almost every educator can agree that there are positives and negatives of using a calculator in the classroom, but what are they exactly? A group of educators, The Room 241 Team, at Concordia University, Portland, composed a list to share with math teachers as they consider how often calculators should be used in their own classroom (Calculators in Math Class: Pros and Cons, 2018).

Pros

- Technological Knowledge – In the society that we live in, technology is always changing. That is why it is so important for students to be using technology and understand how to use it. Whether it is a simple handheld calculator or an application on an iPad or Chromebook, it can benefit students and their understanding of how to use it. Also, calculators are a tool that students will use all throughout their lives from setting a budget, doing their taxes, and other tasks throughout their life.
- Enjoyment – For many students, mathematics is very tedious and not a subject that they enjoy. Calculators can make it a little bit more enjoyable for those students which can lead to a much more positive classroom environment.
- Accuracy – Calculators are an extremely fast and accurate tool for students. They can be used very effectively for checking answers. Students need to learn that calculators are not just for finding answers, but they are a tool to help them understand how to find the answer.

Cons

- Complacency – While accuracy is a pro for calculator use, it can also be a con as students can become complacent and simply use calculators to find answers without knowing how to do them by hand. This is where teachers need to be careful of how much calculator use is allowed so that students are still developing their mathematical skills.
- Cheating – The capabilities of some calculators, such as graphing calculators are fantastic tools, but they can also lead to cheating. For instance, you are able to store notes on some that can be used as cheating for the students. Teachers have to strictly enforce what students can and cannot be doing on the calculator and check on the students often.

- Cost – There are many calculators that are cheap and can be bought easily for under $10. However, more advanced calculators, such as graphing calculators, can be quite expensive. Those calculators are often times more than $100. It may be difficult for some students to afford these tools, that is why it may be beneficial in having a class set of calculators.

Views on Calculator Use in the Classroom

The National Council of Teachers of Mathematics (NCTM) is an organization that was founded in 1920. It is currently the world's largest mathematics education organization in the United States and Canada. As an organization, they strive for high level mathematics teaching and learning for every student. They released their position on calculators and their use in the classroom (Index, n.d.):

> Although calculators—from simple four-function versions to programmable graphing models— are used routinely outside school for a variety of purposes, their specific use in the mathematics classroom must be selective and strategic (NCTM, 2015), with attention to how such a tool will support and advance learning. More important, the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations. Emphasis and implementation are the critical issues.

While NCTM understands the use of calculators in the non-scholastic atmosphere, they recognize the need for students to develop number sense and tools to solve mathematical problems without a calculator.

Another view of calculator use in the classroom comes from Dr. Kevin Kreider who is Chair and Professor of Applied Mathematics at the University of Akron. I recently conducted an interview with him to see what his opinions are on the use of calculators for both high school and college (Kreider, 2019).

Q: In your courses that you have taught at the University of Akron, have you allowed students to use calculators? If so, which courses?

A: In courses below Calculus III, the department policy is to not allow calculators on exams. In upper level courses, in particular Differential Equations, I allow students to use calculators; in many cases, they are required.

Q: What led you to make the decision to allow or not allow students to use calculators?

A: The department policy is guided by 2 issues:

1) Our goal is to teach students how to think mathematically and approach problems with analytic thinking processes; over-reliance on calculators detracts from the learning process.

2) When proctoring exams, it is difficult to distinguish between calculators and mobile devices; there are apps where you can photograph a problem and you get an answer with intermediate steps provided. Good in real life, bad in the classroom.

Q: Have you found that students are too reliant on calculators?

A: Yes, in the introductory level courses, many students have no 'numeracy' (analog to 'literacy') – they have no sense of the magnitude of answers to problems, they cannot estimate quantities, they are inept at algebraic manipulations whether they involve numbers or symbols because they don't have to practice on paper.

Q: If students were not allowed to use calculators in high school, besides mathematics that requires it, do you think students would adapt to their college courses better?

A: Yes, our dream is to coordinate curriculum with all high schools, and that continuity would improve student performance; calculator use is just a part of that. Without extensive use of calculators, students would be more adept at reasoning. Having said that, it is still vital for students to learn to use technology, because they will need it when they join the workforce. The issue is how to teach them. Currently, calculators are used to teach mathematical content, and I think that is counterproductive. Ideally, a course would have 2 components – the main course where they learn content analytically, and a co-requisite course where they learn to use technology.

Q: Do you believe it is more beneficial for students to learn more topics with easier problems, or less material with more difficult problems?

A: Less material with more difficult problems. Generally, the goal of the course is to teach students how to think in a certain way; calculations are a means to that end rather than the end goal itself. A large number of easy problems doesn't develop reasoning skills.

Q: Do you think calculators should be allowed in high school math? Why?

A: Definitely, calculators should be allowed in high school. I would like to see them used differently, as a secondary resource rather than a primary one, but students definitely need to learn how to use technology.

Hearing what Dr. Kreider had to share about calculator use at the University of Akron and in high schools was very interesting. The university's stance of no calculators for math courses Calculus III and below, forces students to think analytically which is one of the goals of

the math department. In terms of over reliance, I can firsthand say that my situation was very similar to what he describes. While I adjusted quickly, it was an adjustment having to think more in a numerical sense instead of relying on the calculators.

On his stance of calculator use in high schools, I feel as though he makes very good points. Not using calculators would help students be able to reason better and solve more difficult problems even if that means less material gets covered. Colleges and universities need to work together more with high schools and have similar policies. It would help students prepare for the next step in their education journey, and with having a technological co-requisite as suggested by Dr. Kreider, it will help them for the real world as well.

Surveys

Over the past few months I have conducted three surveys to understand the views educators have on calculator use. I reached out to numerous teachers and districts across northeast Ohio and got responses from 18 full time mathematics teachers from urban, rural, and suburban districts. This survey questioned their experience with calculators as a student, the use of calculators in their classroom that they teach, and their opinions about calculator use. Another survey that I conducted questioned students of all majors from several universities about their experience with calculators in both high school and college and how it has impacted their learning. I received 42 responses from this survey. For the last survey, I had 87 students from Marlington High School in Alliance, Ohio, answer questions about their current use of calculators in the high school classroom and how they feel calculators have impacted their learning experience thus far. All three of these surveys provided interesting data and several of the results are provided below.

Survey of High School Teachers

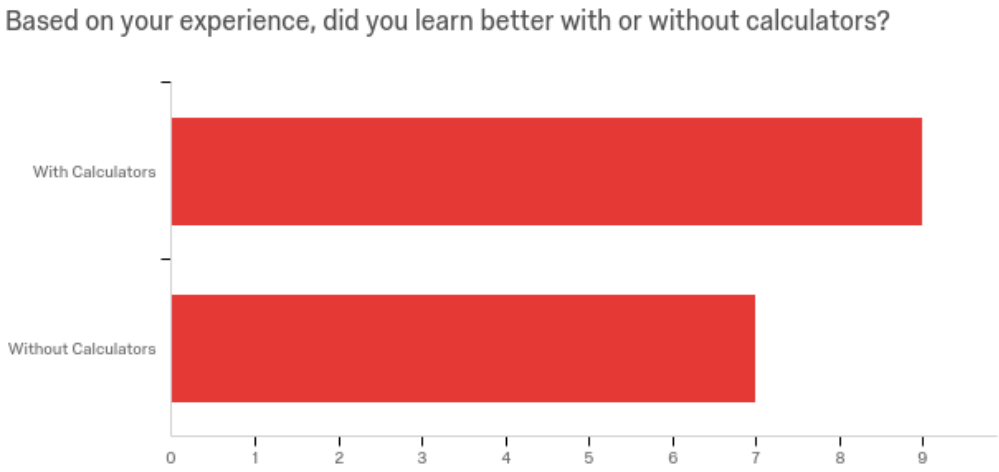

Figure 4: Teacher's experience learning with calculators

- When asked are students allowed to use calculators in their class that they teach and if so which type, not one teacher responded that they do not allow calculators. Those who responded that they teach or have taught Algebra 2, Geometry, Pre Calculus, Calculus, or Statistics let students use either a basic or scientific calculator. While for those who responded teach or have taught Algebra 1, only 1 does not allow scientific calculators while the rest do.

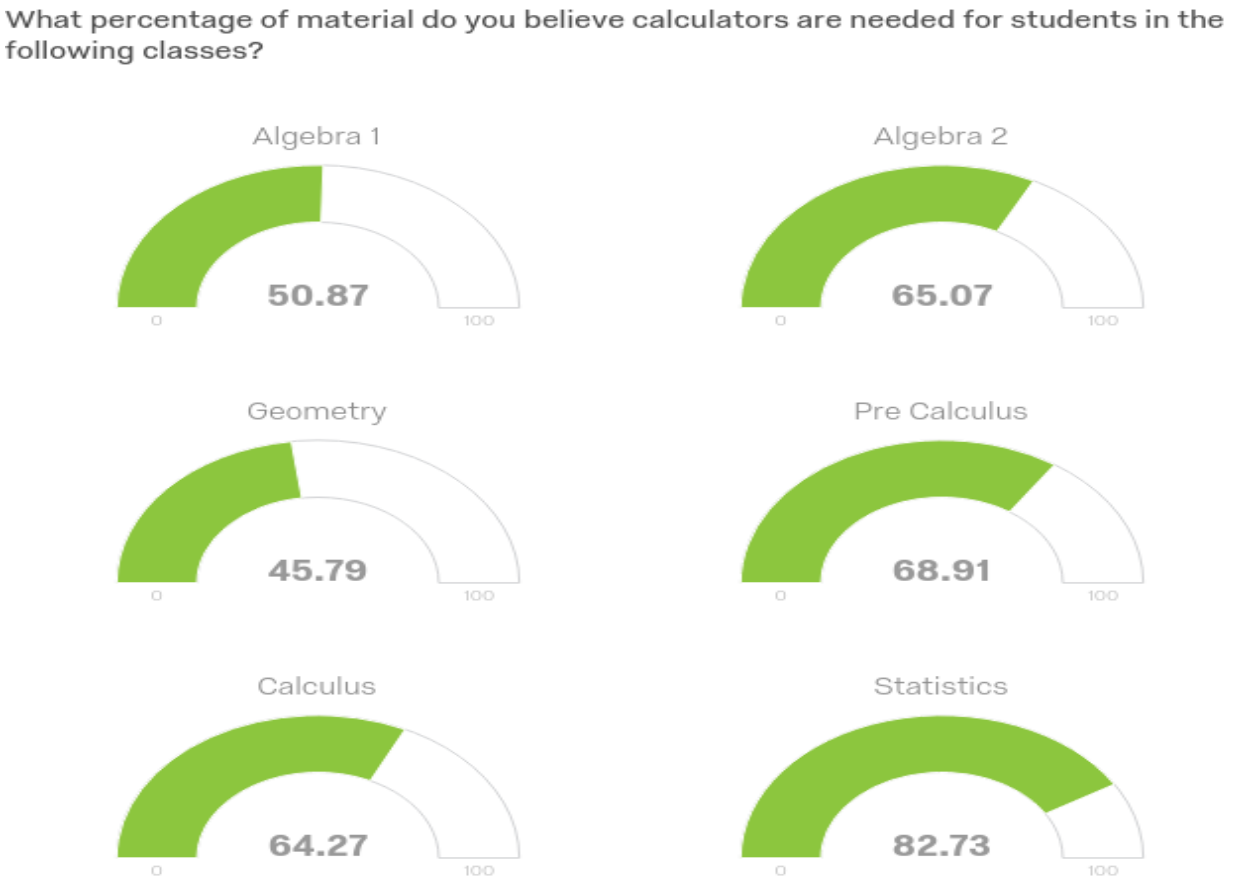

Figure 5: Mean of teacher's responses on the percent they believe calculators are needed in the shown classes

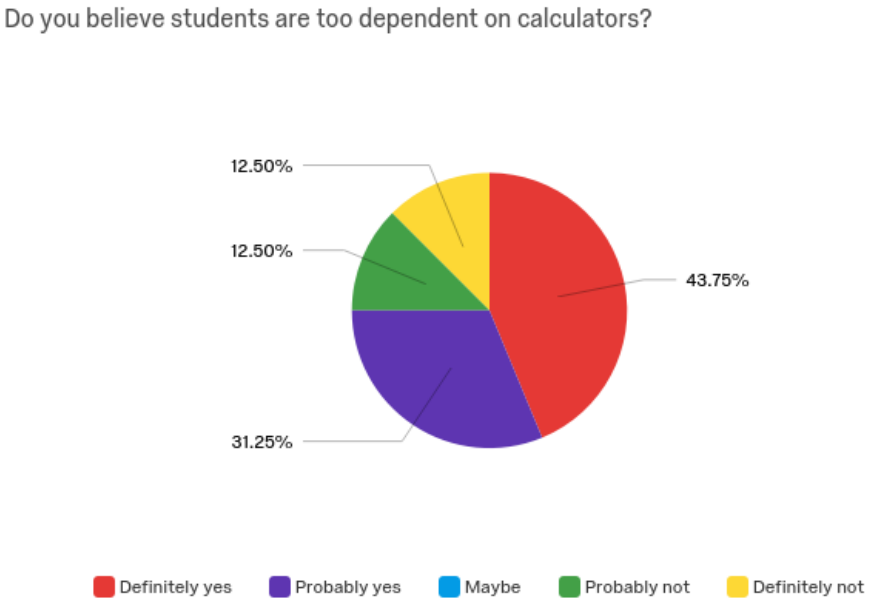

Figure 6: Teachers response on calculator dependence

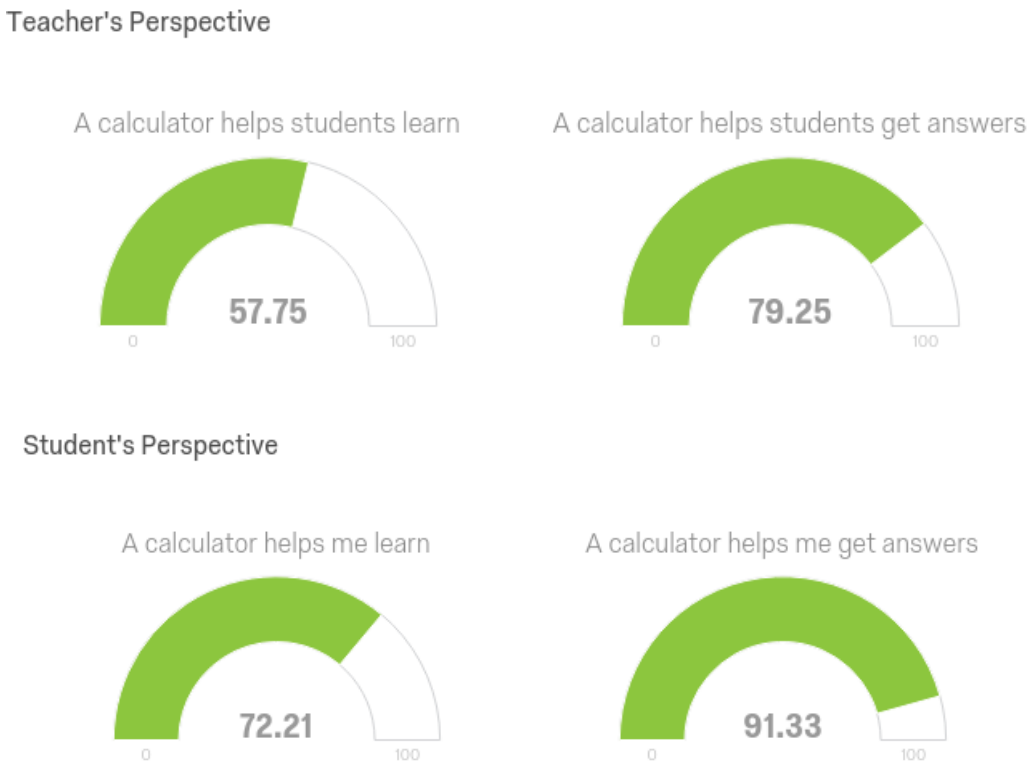

Figure 7: Mean of teacher's views on calculator use and how they believe students view calculator use

Additional comments:

- *"With the massive achievement gaps that some students have, graphing calculators help to address those gaps."*
- *"The use of calculators is only as good as the ability to accurately enter data and interpret answers. Students do not consistently have access to calculators (nor fully understand how to use them), so at times, the calculator is little more than a crutch.*

Greater emphasis needs to be put on number sense, mental math and estimation/approximation at the high school level."

- *"Students need to learn that the calculator can be used for higher level concepts. They should not have to use the calculator for basic "easy" math. Also at the high school level, the calculator can do the basic math to allow for the analysis and better understanding of the harder concepts."*
- *"They should be used to help speed up calculation processes but students do become too dependent on them."*
- *"Students need to be able to do basic facts without a calculator; multiplication facts, division facts, adding integers, simple fractions. Students are too dependent on calculators and they often do not understand what they are putting into the calculator. They also need to know and understand order of operations to use a calculator correctly."*
- *"I think that the potential is there for students to learn by using the calculator but I don't think that they are usually used for this purpose. Students do need value learning basic math facts because they use the calculator as an excuse to not learn them."*
- *"There are students with special needs who depend on calculators to access grade level material. They would otherwise be caught up in struggling to calculate the function instead of learning the concept."*

Analysis: For the most part, it seems as though the majority of teachers feel as though students are too dependent on calculators in their class, and in some ways, the over dependence is impacting their potential to maximize their learning. From the additional comments it seems as though most teachers think that the way calculators are used could be

more beneficial in a different way, but they don't know what that way could be. I think that is exactly what Dr. Kreider was talking about when he emphasized that universities and high schools should work together on how to implement calculator use. I feel as though that could help students immensely.

Survey of College Students:

Were you allowed to use calculators in the following courses in college?

#	Field	Yes	At Times	No	Did not take in college
4	Calculus 3	17.07% 7	14.63% 6	39.02% 16	29.27% 12
1	Pre Calculus	21.43% 9	9.52% 4	9.52% 4	59.52% 25
2	Calculus 1	2.44% 1	7.32% 3	43.90% 18	46.34% 19
3	Calculus 2	9.76% 4	7.32% 3	58.54% 24	24.39% 10
6	Statistics	37.50% 15	5.00% 2	10.00% 4	47.50% 19
5	Differential Equations	12.20% 5	4.88% 2	48.78% 20	34.15% 14

Showing rows 1 - 6 of 6

Figure 8: Calculator use in college courses

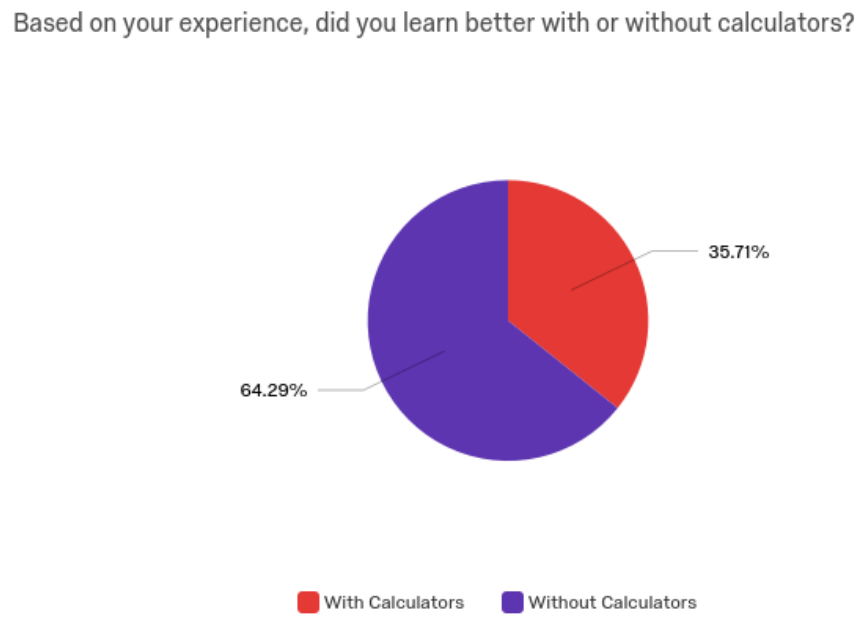

Figure 9: College student's views on if they learned better with or without calculators

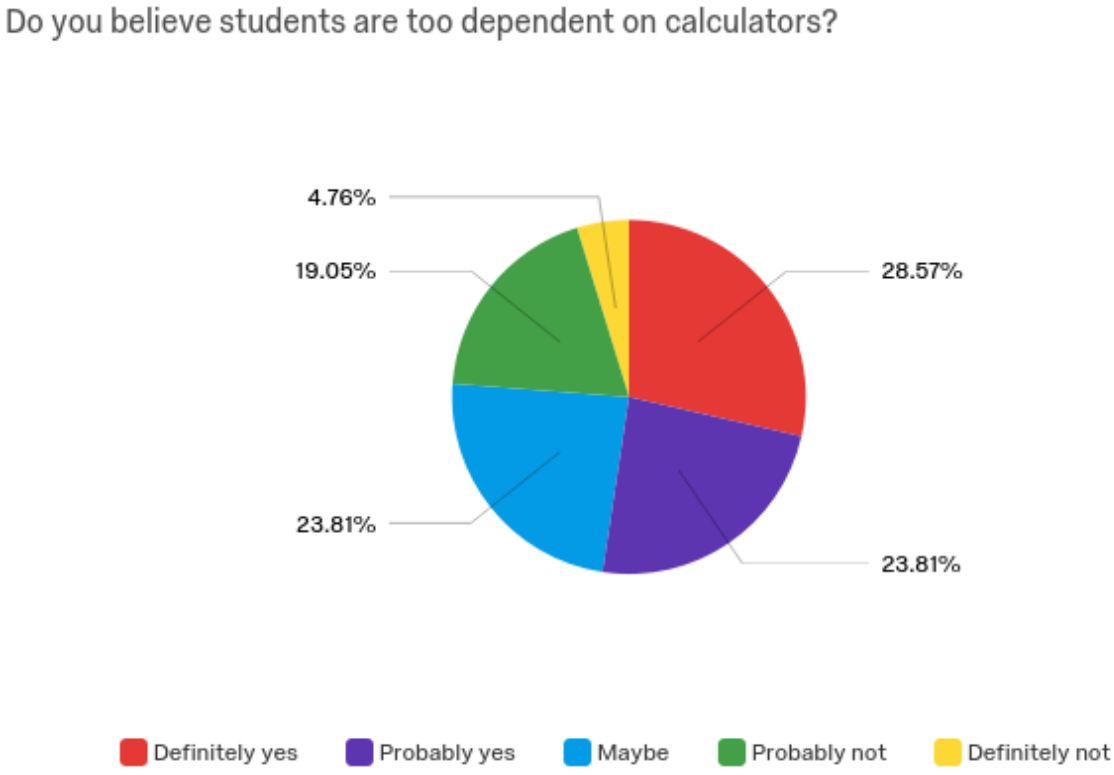

Figure 10: College student's views on calculator dependence

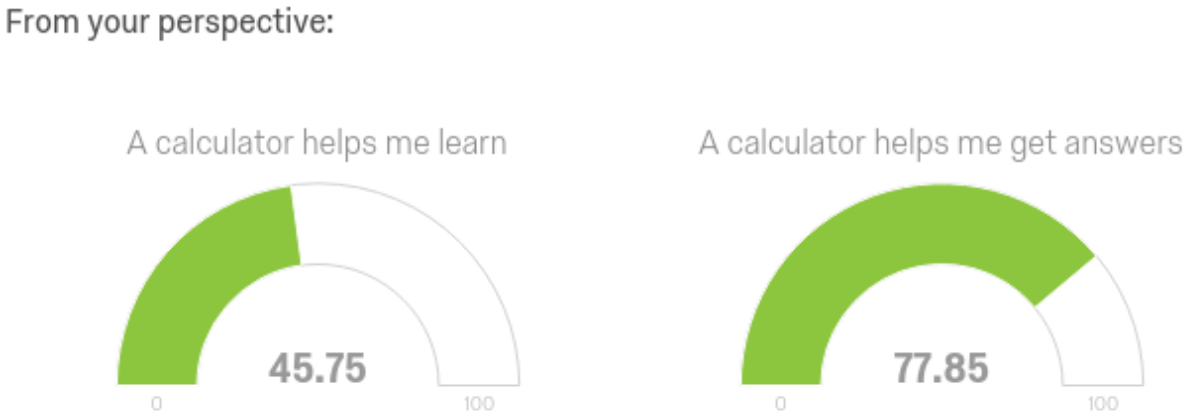

Figure 11: Means of college student's views on use of calculators

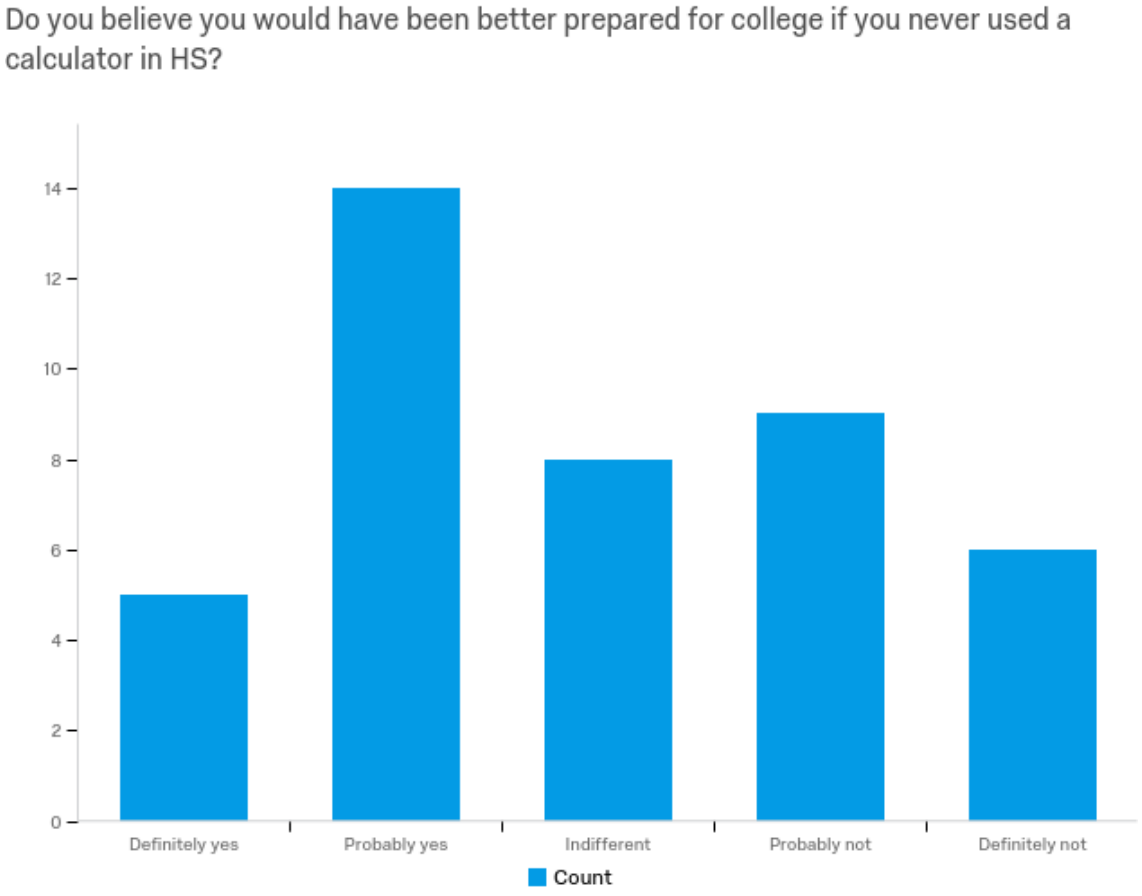

Figure 12: College student's opinion on how calculators effected their college preparedness

Analysis: The majority of college students in this survey have taken a mathematics course where they haven't been allowed to use calculators in college. While in high school, all but two said they were allowed to use them. It is interesting to see that nearly 65% of students believe that they learn better without using a calculator. Yet their opinions seemed to be split on if not having a calculator in high school would have prepared them better for college.

Survey of High School Students:

What type of calculator have you been allowed to use in the following classes?

#	Field	Basic Calculator	Scientific Cacluclator	Either	No Calculator	Never Taken
1	Algebra 1	44.19% 38	24.42% 21	29.07% 25	2.33% 2	0.00% 0
2	Algebra 2	16.28% 14	41.86% 36	41.86% 36	0.00% 0	0.00% 0
3	Geometry	15.12% 13	47.67% 41	37.21% 32	0.00% 0	0.00% 0
4	Pre Calculus	5.88% 5	54.12% 46	29.41% 25	0.00% 0	10.59% 9
5	Calculus	2.47% 2	20.99% 17	9.88% 8	0.00% 0	66.67% 54
6	Statistics	2.50% 2	26.25% 21	5.00% 4	0.00% 0	66.25% 53

Showing rows 1 - 6 of 6

Figure 13: Calculator use in high school classes

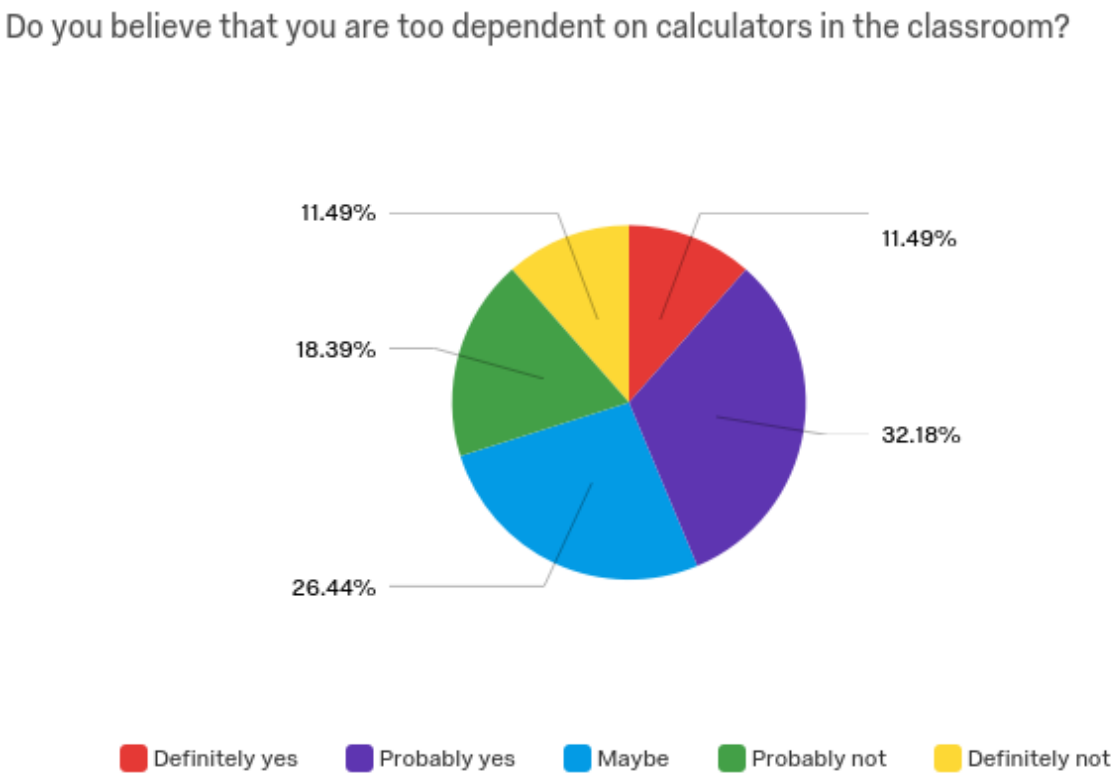

Figure 14: High school student's view on calculator dependence

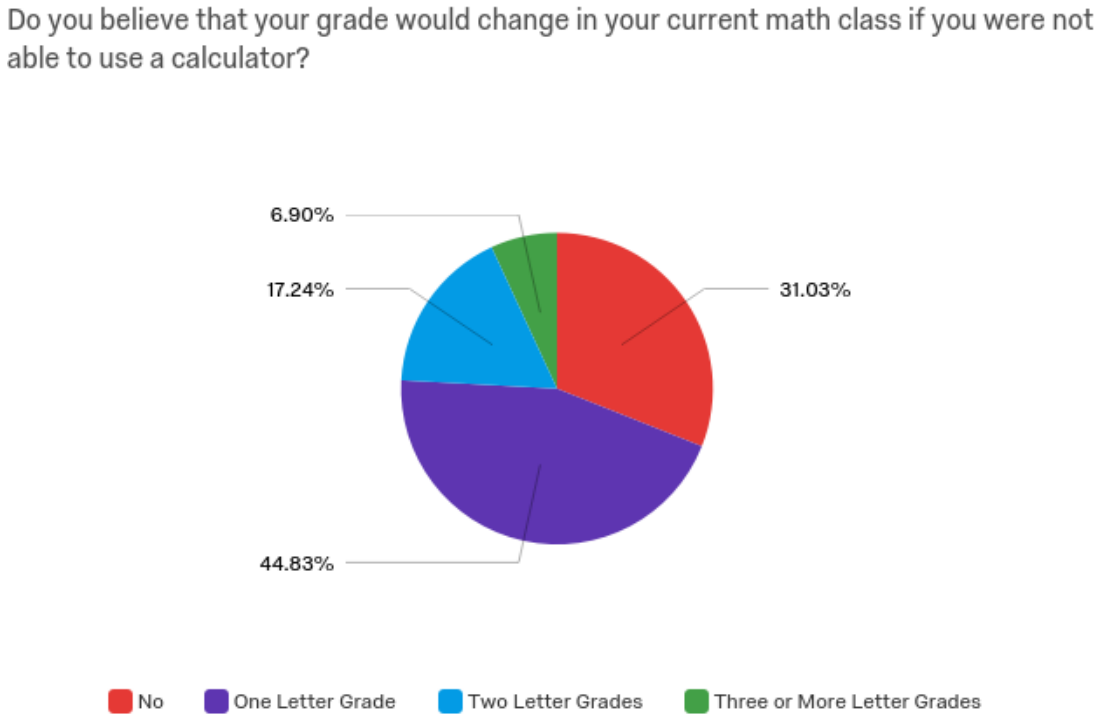

Figure 15: High school student's view on grade impact

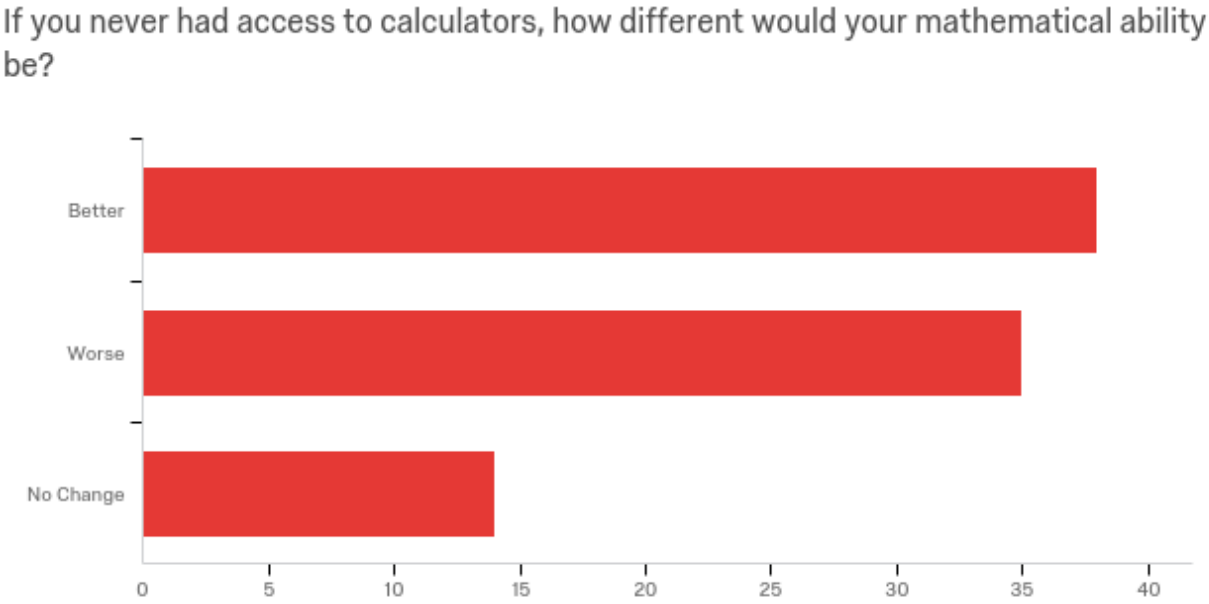

Figure 16: Opinion of high school students and how calculators have affected their math ability

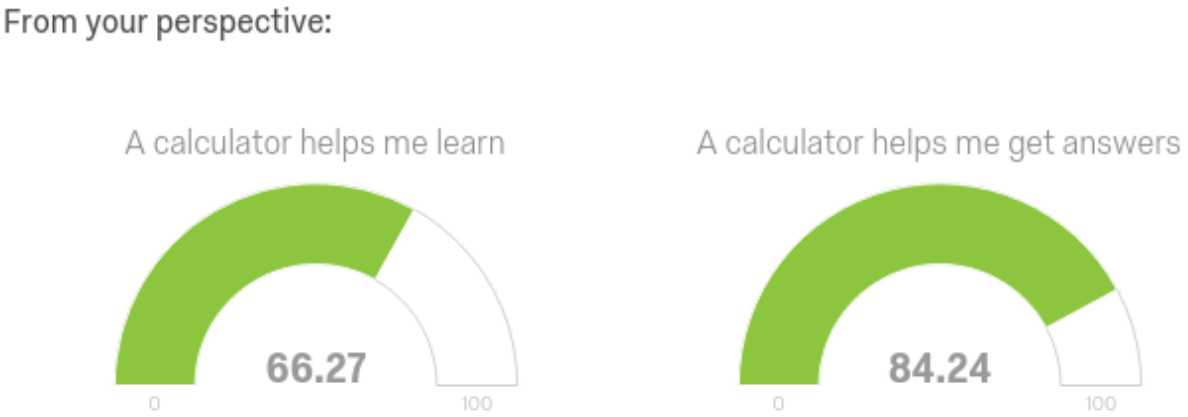

Figure 17: Mean of high school student's view on use of calculators

Analysis: I found the results of this survey to be very interesting. Almost every student has been allowed to use a calculator in their high school classes. While students are about split on whether or not they are too dependent on calculators, most feel as though if they were not allowed to use calculators that their grade would change. Of course, there is some math that is needed to be done with the aid of a calculator, but I was not expecting as many to say they think their grade

would change. Even more to my surprise, the majority of students believe that their mathematical ability would be better if they never had access to calculators.

Calculator Use in Other Countries

James Tarr, Department of Curriculum and Instruction at the University of Missouri, led a study which used data from the Third International Mathematics and Science Study (TIMSS) to determine trends in calculator use for students in the countries of the Unites States, Portugal, and Japan. These countries were chosen as they scored significantly different from the United States average score. The United States average score was a 500, while Japan had an average of 605 which was third highest of the countries that participated, and Portugal averaged a 454 which was the sixth lowest of the countries that participated. The study focused on the data from 13-year-old students or those who were in the eighth grade. It focused on the data provided from both teachers and students about how often calculators were used and in which ways they were used (Tarr, 2010).

In each country, students reported the frequency in which calculators were used in the classroom. In the United States, almost 68% of eighth graders reported that they use calculators "almost always" or "pretty often". Portugal students reported similarly with 69% of their students reporting the same thing. Meanwhile in Japan, only 4% of students reported that they used calculators in the classroom "almost always" or "pretty often." Also, almost 75% of Japanese students said that they never use calculators during math lessons in class (Tarr, 2010).

Teachers from all three countries gave responses on which ways calculators were used in their classrooms during their lessons. The table can be seen below (Tarr, 2010):

Country	Checking Answers	Tests and Exams	Routine Computation	Solving Complex Problems	Exploring Number Concepts
Japan	0.04	0.01	0.14	0.11	0.16
	(0.02)	(0.00)	(0.04)	(0.03)	(0.05)
U.S.	1.52	1.42	1.96	2.17	1.71
	(0.09)	(0.11)	(0.11)	(0.09)	(0.09)
Portugal	2.20	1.40	2.12	1.89	1.63
	(0.10)	(0.08)	(0.09)	(0.10)	(0.09)

Note. Teachers' responses were receded in the following manner: 0 =*never or hardly ever*, 1 =*once or twice per month, 2 = once or twice a week*, 3 =*almost every day.* Standard errors are reported in parentheses.

Figure 18: Mean Responses of Teachers' Reported of Ways Calculators Used in Mathematics Lessons

Based on the reports of student's frequency of calculator use, calculators do seem to be related to achievement in the United States; however, it is not significant. Meanwhile in Portugal, it is negatively associated by a very small margin. In Japan there was a significantly negative relation between achievement and calculator use (Tarr, 2010). While it is a positive that

there seems to be a positive trend in the United States with achievement and test scores, the average score is still very far behind that of the average student in Japan who, as students reported, rarely use calculators. Of course, there are cultural differences between the country's curriculum, standards, and environment, but Japan is one of the most technologically advanced countries in the world. With students reporting that they rarely use calculators makes one wonder if less countries would use calculators would their scores be higher?

Summary

Calculators have come a long way since the very first calculating tools were used. They have countless features and make our lives much easier, but is that a good thing? Of course, in the professional and non-scholastic world, it is hard to see any disadvantages as they provide accurate results, save time, and ease the stress of many. However, do they potentially hurt student learning in the classroom? It is a question that should be raised especially when Japanese students report that they rarely use calculators in the classroom, and they score significantly higher as a country in mathematics compared to American students.

High school teachers seem to agree that students are at least a little bit too dependent on calculators and that they lack some number sense because of the way that they use calculators. The majority of college students stated that they believe that they learned mathematics better without being allowed to use a calculator compared to when they were allowed to. Even the majority of high school students who were surveyed believed that they would have a higher mathematical ability had they never been allowed to use calculators in class. If all three groups believe that calculators are potentially holding back their ability to learn in some way, why are they as commonly used as they are? Of course, calculator use varies from course to course, but

as Dr. Kreider mentioned, they should be used as a secondary resource not a first. This way allows students to hopefully learn material first and then use that knowledge and apply it with technology.

Mathematics is one of the most important skills students learn in the education system. With that being said, educators should do their best to help students maximize their learning potential. Is that with calculators being used the way they typically are in the classroom or is it adapting and pushing students to think and solve problems more independently while using calculators as a secondary tool? It is hard to say that one way is the correct way with school environments, achievement gaps, and student's previous knowledge playing such huge factors. But is the duty of current and future educators to explore this and help the future generations reach their full potential.

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